APPENDIX

BAYESIAN REGRESSION ANALYSIS FOR YIELD-RESPONSE EXPERIMENTS

A. Nature of the Analysis

The purpose of this appendix is to describe how the observations obtained from a dose-response experiment can be modeled using Bayesian regression analysis. A Bayesian approach is necessary to provide inputs in a form appropriate for making a decision regarding an economically efficient level of environmental regulation.

Consider the following "simple normal linear" regression model [Zellner (1971)]:

$$Y_k = \alpha + \beta X_k + \varepsilon_k \tag{A.1}$$

$k = 1, 2, \ldots, n$, with the error term (ε_k) being independently normally distributed with zero mean and constant variance σ^2, $[N(0,\sigma^2)]$. Here the X_k denote the level of pollutant applied to the kth plot of the experiment and Y_k denotes the corresponding observed crop yield for that plot. In keeping with the Bayesian approach, the parameters, α, β, and σ^2 of the regression model are viewed as random variables, rather than as unknown constants.

Assume for purposes of illustration that no prior information is available concerning the parameters of the regression model. In particular, not even the sign of the slope β of the regression (or yield response) function is assumed to be known. We are allowing for the possibility, _a priori,_ that crop yield Y and pollutant concentration X have a positive association. Formally, it is mathematically convenient to assume that α, β, and $\log \sigma$ are uniformly and independently distributed, _a priori_. Such a "diffuse" prior probability distribution has probability density function

$$p\ (\alpha,\ \beta,\ \sigma) \propto \frac{1}{\sigma} \tag{A.2}$$

$$-\infty < \alpha < \infty,\ -\infty < \beta < \infty,\ 0 < \sigma < \infty.$$

We wish to estimate the mean yield Y_h corresponding to setting a pollutant concentration standard X_h. For convenience, express this mean yield Y_h as a fraction, T_h say, of the mean yield Y associated with the current pollutant level X_o; that is,

$$T_k = Y_h/Y_o. \tag{A.3}$$

The yield ratio (3) can be reexpressed as

$$T_h = 1 + \beta(X_h - X_o)/Y_o. \tag{A.4}$$

It is assumed that the current mean yield Y_o is known, so that by (4), the yield ratio T_h is simply a linear transformation of the slope β of the yield-response function.

We now obtain the posterior probability distribution of the yield ratio T_h given the sample of n observations $\{(X_k, Y_k): k = 1, 2, \ldots, n\}$ generated by the experiment. The slope β of the yield-response function has posterior probability distribution of the Student t form; specifically,

$$t_{n-2} = \frac{(\beta - \hat{\beta})}{s(\hat{\beta})} \tag{A.5}$$

is a random variable having the Student t distribution with n-2 degrees of freedom. Here

$$\hat{\beta} = \frac{\sum_{k=1}^{n} (X_k - \overline{X})(Y_k - \overline{Y})}{\sum_{k=1}^{n} (X_k - \overline{X})^2}, \text{ and } \hat{\alpha} = \overline{Y} - \hat{\beta}\,\overline{X}, \tag{A.6}$$

and

$$s^2(\hat{\beta}) = \frac{s^2}{\sum_{k=1}^{n} (X_k - \overline{X})^2} \tag{A.7}$$

with

$$s^2 = \frac{1}{n-2} \sum_{k=1}^{n} (Y_k - \hat{Y}_k)^2, \text{ and } \hat{Y}_k = \hat{\alpha} + \hat{\beta}X_k. \tag{A.8}$$

From the posterior distribution of the slope β of the yield response function, it follows that the posterior distribution of the yield ratio T_h is also the Student t form; specifically,

$$(T_h - \hat{T}_h)/s(\hat{T}_h) \tag{A.9}$$

is a random variable having the Student t distribution with n-2 degrees of freedom. Here

$$\hat{T}_h = 1 + \hat{\beta}(X_h - X_o)/Y_o \tag{A.10}$$

and

$$s(\hat{T}) = s(\hat{\beta}) \mid X_h - X_o \mid /Y_o. \tag{A.11}$$

This result concerning the form of posterior probability distribution of percent yield reduction, T_h, enables us to make probability statements (e.g., to determine the probability that T_h is greater than a certain specified value, given our sample). In particular, the posterior probability distribution of T_h will be used to compute expected benefits in the decision-making problem of setting a standard on pollutant concentration X.

B. Example: cotton-ozone data

We now demonstrate the application of the Bayesian regression methodology for estimating yield response functions. Data are taken from an agronomic experiment involving cotton plants which were exposed to different ozone concentrations. The 12 pairs of observations (i.e., n = 12) of mean seasonal ozone concentration X (ppm) and cotton yield Y (grams) per plot are listed in Table A1.

Employing the simple normal linear regression model with the diffuse prior probability distribution, (A.2), assumed for the parameters α, β, and σ, the following statistics were obtained:

$$\alpha = 1098.39 \text{ g}, \quad \beta = -3707.99 \text{ g/ppm}, \quad s(\hat{\beta}) = 288.52 \text{ g/ppm} \tag{A.12}$$

The slope, β, of the cotton-ozone dose-response function has posterior probability distribution of the Student t form; namely,

$$t_{(10)} = [\beta - (-3707.99)] / 288.52 \tag{A.13}$$

has a Student t distribution with n-2 = 10 degrees of freedom. Figure Al shows the posterior probability density function for β. Although the prior distribution allowed for the possibility that β is positive (i.e., a positive association between cotton yield and ozone concentration), *a posteriori* the probability that β is positive is virtually zero (in fact, smaller than 0.005%).

We now wish to estimate the percent yield ratio T_j for various levels of ozone concentration X_h, relative to a current mean cotton yield of Y_o = 838.83 g corresponding to a current ozone concentration of 0.07 ppm. For convenience, the value of Y_o was obtained by using the regression coefficient estimates α and β, whereas the form of the posterior distribution of the yield ratio T_h, (A.9), requires that Y be known. Taking the case of ozone concentration X_h = 0.06 ppm, (A.10) and

TABLE A1

Mean Seasonal Ozone Concentration and Cotton Yield by Plot

Ozone is in ppm. Cotton yield is in grams.

Plot Number k	Ozone Concentration X_k	Cotton Yield Y_k
1	0.018 ppm	1030
2	0.032	1030
3	0.046	988
4	0.043	936
5	0.070	781
6	0.073	868
7	0.113	633
8	0.107	600
9	0.144	647
10	0.138	573
11	0.179	409
12	0.186	456

Figure A1

POSTERIOR DISTRIBUTION OF COTTON SLOPE

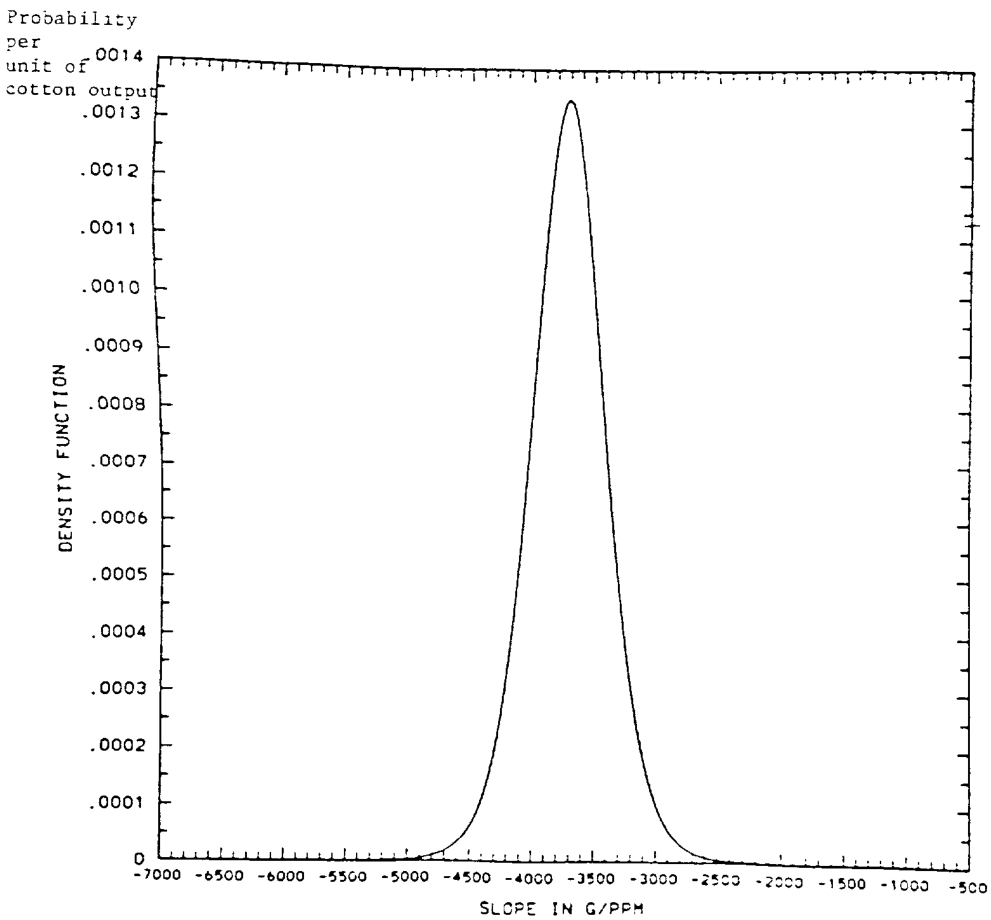
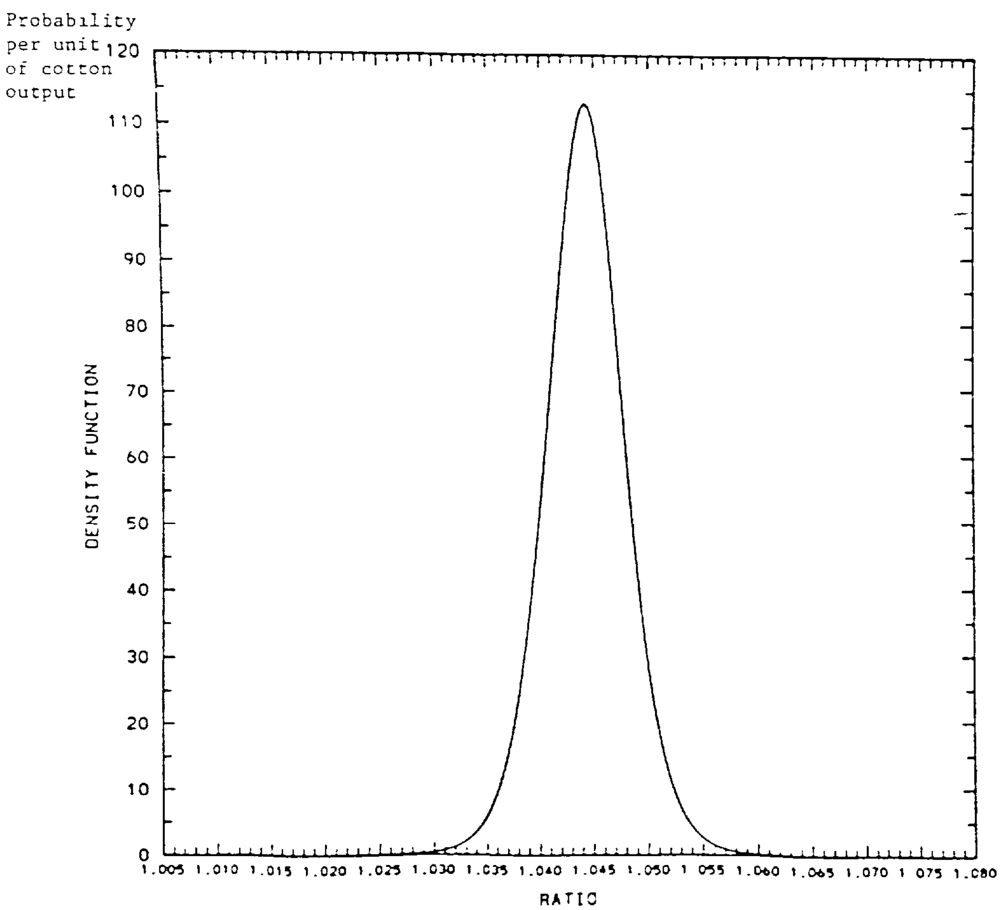


Figure A2

POSTERIOR DISTRIBUTION OF COTTON YIELD RATIO



(A.11) give:

$$\hat{T}_h = 1.0331,\ s(\hat{T}_h) = 0.003440. \tag{A.14}$$

Thus the standardized yield ratio

$$(T_h - 1.0442)/(0.003440) \tag{A.15}$$

has a Student t distribution with 10 degrees of freedom. Figure A2 shows the posterior probability density function for T_h. We note, for instance, that T_h falls between 1.0365 and 1.0519 with a 95% chance _a posteriori._

REFERENCES

[1] We thus disregard the abundant sources of uncertainty residing in the economic propositions and empirical applications that support control benefits assessments.

[2] On the other hand, Smith and Vaughn (1980) and Kopp and Smith (1982) provide some empirical support on the cost side for the premise. In their studies of the costs of pollution control in the iron and steel industry, they found their cost estimates to be very sensitive to the engineering details embedded in their models.

[3] See Crocker (1982) for more details. Adams, *et al*. (1982) employed a price endogenous, quadratic programming model to examine the economic impact of ambient oxidants upon the 1976 production of 14 annual crops in four southern California subregions. For all but two the 56 possible region-crop combinations, the differences between estimated and actual levels of crop production were substantially less than ± 10 percent. In 29 of the 56 combinations, the predicted percentage yield change inclusive of the economic reactions differed from the triggering percentage yield change by a factor of 2 or more. Many, perhaps most, of these latter differences are accounted for by the propensity of farmers to take advantage of changes across crops in most favorable production opportunities. The errors in predicting ultimate yield responses that neglect of farmers' economic reactions will introduce can be rigorously shown to be inversely dependent on the absolute curvature of the production possibility surfaces and the price flexibility of crop supplies.

[4] The pollution exposure (dose) in each of the yield response expressions was measured as a seven-hour seasonal mean concentration of ozone. The seven-hour period is from 9:00 a.m. to 4:00 p.m., the period in which stomata1 activity and hence plant sensitivity to pollution is greatest. In order to transform the mean seven-hour dose to the same basis as the SNAAQS, ambient ozone is assumed to be log-normally distributed. Thus, for example, a seasonal seven-hour concentration of .07 ppm is treated as being a SNAAQS concentration of 14 ppm.

[5] In accordance with expression (2) of the text, the expected payoffs of the alternative standards are the E[W(i)] - E[W(o)] less the costs of implementing the alternatives. USEPA's Office of Air Quality Planning and Standards (1979) has estimated the costs of implementing a range of alternative ozone standards similar to those we consider at $3 billion to $9 billion annually. Crocker (1982) suggests that total agricultural benefits from all classes of improved air quality may not exceed 10-20 percent of total air pollution control benefits. *If* cost

responsibilities are assigned to agriculture in accordance with its supposed share of these total benefits, then the expected payoffs for the 0.10 ppm and the 0.08 ppm standards are positive. However, about half the gain in surplus associated with going from the 0.12 ppm standard to the 0.08 ppm standard is due to the estimated increase in corn yields. We have recently experimented with a quadratic form for the corn yield response function and have found that yield responses and consequent changes in economic surplus are somewhat lower in absolute magnitude than the corn surplus used to arrive at Table 4. In particular, with a quadratic yield response function for corn, Table 4 becomes:

Ambient Standard a_i	Expected Surplus E[W(i)]	Change in Expected Surplus E[W(i)] - E[W(o)]
0 (0.12 ppm)	51.3	---
1 (0.10 ppm)	54.6	3.3
2 (0.08 ppm)	57.8	6.5
3 (0.14 ppm)	47.5	-3.8

More significantly, the density functions for the quadratic version of Figure 2 now display no overlap. This suggests that biological model uncertainty may be as important a factor as sample size (precision) in the role that yield response information plays in benefit-cost analysis.

[6] The policymaker would have to possess a loss function putting extremely heavy emphasis on Type I error in order to be very concerned with the overlap between the 0.10 and 0.08 surplus distributions for corn and wheat.

[7] See Adams and Crocker (1982) for detail on the features of these differential yield responses that are of particular interest to economists. If research resources are limited, the decision problem of which crops are deserving of additional yield response observations resembles a portfolio problem. The crops are the kinds of securities and the observations are the number of units of each kind of security to be held.

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